Uitwerking 24-6-'98

① Tegenvoorbeeld. Twee inprodukten in $\mathbb{R}^2$:
$\langle , \rangle_1 = x_1y_1 + x_2y_2$ en $\langle , \rangle_2 = 2x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2$
Stel $L = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix}$, L is hermitisch m.b.t. $\langle , \rangle_1$
$\langle Lx, y\rangle_1 = ax_1y_1 + x_2y_2 = \langle x, Ly\rangle$ $\quad \left(L\binom{x_1}{x_2} = \begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix}\binom{x_1}{x_2} = \binom{ax_1}{x_2}\right)$
maar niet hermitisch m.b.t. $\langle , \rangle_2$, want
$\langle Lx, y\rangle_2 = 2ax_1y_1 + ax_1y_2 + x_2y_1 + x_2y_2 \neq \langle x, Ly\rangle_2 =$
$2ax_1y_1 + x_1y_2 + ax_2y_1 + x_2y_2$. Dus als een operator bij één inprodukt hermitisch is, hoeft hij dat niet te zijn bij een ander.

② Unitair, dus $a_{11}^2 + a_{12}^2 + a_{13}^2 = 1$
$a_{21}^2 + a_{22}^2 + a_{23}^2 = 1$
$a_{31}^2 + a_{32}^2 + a_{33}^2 = 1$
Dit zijn allemaal kwadraten van reële getallen, a_{ij}^2 is dus maximaal 1, m.a.w. $|a_{ij}| \leq 1$

③ $h = g \circ f = \begin{pmatrix} 1 \\ \cos(x^2+y+y^2+\sin x) \end{pmatrix}$

$$Dh = \begin{pmatrix} 0 & 0 \\ -(2x+\cos x)\sin(x^2+y+y^2+\sin x) & -(2y+1)\sin(x^2+y+y^2+\sin x) \end{pmatrix}$$

$$h'(\pi,0) = Dh(\pi,0) = \begin{pmatrix} 0 & 0 \\ -(2\pi-1)\sin\pi^2 & -\sin\pi^2 \end{pmatrix}$$

④ 1. $Df = \begin{bmatrix} 0 & 2ye^{y^2} & 0 \\ -y\sin x & \cos x & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 2. $Df(0,0,0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. f is continu in (0,0,0), Df is continu in (0,0,0), want e^{y^2}, $2ye^{y^2}$, $y\cos x$, $-y\sin x$, $\cos x$ zijn allemaal continu in $(0,0,0) \Rightarrow$
f is differentieerbaar in (0,0,0)

⑤ Theorem 9.2 D'Alembert's solution of the wave equation
$f(0,x) = F(x)$, $\frac{\partial}{\partial t} f(0,x) = G(x)$
Dan voldoet de volgende $f(x,t)$ aan de golfvergelijking
$$f(x,t) = \frac{F(x+t)+F(x-t)}{2} + \frac{1}{2}\int_{x-t}^{x+t} G(s)\,ds = x+t+2$$

⑥ Eis voor stationair punt: $\frac{\partial}{\partial x} f = 0$, $\frac{\partial}{\partial y} f = 0$, $\frac{\partial}{\partial z} f = 0$

$\frac{\partial f}{\partial x} = 2x + 3y + 128x^3y^5 \sin e^z$

$\frac{\partial f}{\partial y} = 2y + 3x + 160x^4y^4 \sin e^z$

$\frac{\partial f}{\partial z} = 2z + 32x^4y^5e^z \cos e^z$

Invullen $(x,y,z) = (0,0,0)$ geeft $\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$ en $\frac{\partial f}{\partial z} = 0$ $\Rightarrow$ stationair punt

$\frac{\partial^2 f}{\partial x^2} = 2 + 384x^2y^5 \sin e^z$

$\frac{\partial^2 f}{\partial y^2} = 2 + 640x^4y^3 \sin e^z$

$\frac{\partial^2 f}{\partial z^2} = 2 + 32x^4y^5e^z \cos e^z - 32e^{2z}x^4y^5 \sin e^z$

$\frac{\partial^2 f}{\partial x \partial y} = 3 + 640x^3y^4 \sin e^z = \frac{\partial^2 f}{\partial y \partial x}$

$\frac{\partial^2 f}{\partial x \partial z} = 128x^3y^5 e^z \cos e^z = \frac{\partial^2 f}{\partial z \partial x}$

$\frac{\partial^2 f}{\partial y \partial z} = 160x^4y^4 e^z \cos e^z = \frac{\partial^2 f}{\partial z \partial y}$

$$H(0,0,0) = \begin{bmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = A \quad \rightarrow \quad \Delta = \det A = -10$$

$\lambda_1 + \lambda_2 + \lambda_3 = 6$

$\Delta = \lambda_1 \lambda_2 \lambda_3 = -10$

Of $\lambda_1 < 0, \lambda_2 < 0, \lambda_3 < 0$ ① of $\lambda_1 < 0, \lambda_2 > 0, \lambda_3 > 0$ ② (1,2,3 willekeurig)

① kan niet want $\lambda_1 + \lambda_2 + \lambda_3 > 0$ $\Rightarrow$ ② $\Rightarrow$ zadelpunt

⑦ $\nabla f = \lambda \nabla g$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= \lambda \frac{\partial g}{\partial x} & 1 &= 2\lambda x \\ \frac{\partial f}{\partial y} &= \lambda \frac{\partial g}{\partial y} & 0 &= 2y \\ \frac{\partial f}{\partial z} &= \lambda \frac{\partial g}{\partial z} & 1 &= 2\lambda z \end{aligned}\right\} \begin{aligned} y &= 0 \\ x &= z \end{aligned}$$

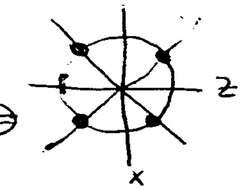
$x^2 + z^2 = 6 \rightarrow 2x^2 = 6 \rightarrow x = \pm\sqrt{3}$

f minimaal in $(-\sqrt{3}, 0, -\sqrt{3})$ en heeft de waarde $-2\sqrt{3}$

$x^2 + z^2 = 6$, onafhankelijk van y, y^2 minimaal bij $y = 0$,

Nu moet $x + z$ minimaal worden. Het punt kan gevonden worden door $x = \pm z$ te snijden met de cylinder

$\Rightarrow$



(8) $\iiint_S \operatorname{div} f_1 - \iiint_S \operatorname{div} f_2 = \iiint_S (\operatorname{div} f_1 - \operatorname{div} f_2) =$

$\iiint_S \left(\frac{\partial}{\partial x}(f_1 - f_2) + \frac{\partial}{\partial y}(f_1 - f_2) + \frac{\partial}{\partial z}(f_1 - f_2)\right)$

maar voor ieder punt (x,y,z) geldt $f_1 = f_2 \Rightarrow$

$\iiint_S \operatorname{div} f_1 - \iiint_S \operatorname{div} f_2 = 0 \Rightarrow \iiint_S \operatorname{div} f_1 = \iiint_S \operatorname{div} f_2$

(9) $\iint_{\mathbb{R}^2} f = \lim_{a\to\infty} \lim_{b\to\infty} \int_{-a}^{a} \int_{-b}^{b} f(x,y)\,dx\,dy$

(10) tja